Dynamics of driven interfaces in algebraically correlated random media

M.-P. Kuittu,[1] M. Haataja,[2] and T. Ala-Nissila[1,3,*]

[1]Helsinki Institute of Physics, P.O. Box 9 (Siltavuorenpenger 20 C), FIN-00014 University of Helsinki, Helsinki, Finland

[2]Centre for the Physics of Materials, Department of Physics, McGill University, Rutherford Building, 3600 rué University, Montréal, Québéc, Canada H3A 2T8

[3]Laboratory of Physics, Helsinki University of Technology, P.O. Box 1100, FIN-02015 HUT, Espoo, Finland
and Department of Physics, Brown University, Providence, Rhode Island 02912-1843

In this work we consider the dynamics of interfaces embedded in algebraically correlated two-dimensional random media. We study the isotropic percolation and the directed percolation lattice models away from and at their percolation transitions. Away from the transition, the kinetic roughening of an interface in both of these models is consistent with the power-law correlated Kardar-Parisi-Zhang universality class. Moreover, the scaling exponents are found to be in good agreement with existing renormalization-group calculations. At the transition, however, we find different behavior. In analogy to the case of a uniformly random background, the scaling exponents of the interface can be related to those of the underlying percolation transition. For the directed percolation case, both the growth and roughness exponents depend on the strength of correlations, while for the isotropic case the roughness exponent is constant. For both cases, the growth exponent increases with the strength of correlations. Our simulations are in good agreement with theory. [S1063-651X(99)04003-9]

I. INTRODUCTION

The problem of interface dynamics in random media has received considerable attention during the past decade. Many different physical problems, such as the pinning and dynamics of flux lines in superconductors [1], dynamics of slow combustion fronts in paper [2–7], and imbibition (paper wetting) [8–11], can be formulated in terms of interfaces propagating in random media. Common theoretical approaches to study these phenomena include lattice models [12] and various stochastic partial differential equations as the equation of motion for the interface, the prototype of which is the celebrated Kardar-Parisi-Zhang (KPZ) type of nonlinear equation [13,12]:

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}|\nabla h(x,t)|^2 + F + \eta. \qquad (1)$$

Here $h(x,t)$ is the height of the interface, F is the driving force, and η denotes the noise at the interface, which in the most general case may exhibit nontrivial correlations in space and time and may also depend explicitly on $h(x,t)$.

In the case where η is a function of $h(x,t)$ and sufficiently short-ranged, the propagating interface may become pinned below a critical value F_c of the driving force. For large values of F ($F \gg F_c$) then, the noise is effectively Gaussian white noise with $\langle \eta(x,t) \rangle = 0$ and $\langle \eta(x,t)\eta(x',t') \rangle = 2D\delta(x-x')\delta(t-t')$. The behavior of lattice models belonging to this so-called thermal Kardar-Parisi-Zhang (TKPZ) universality class is well understood for one-dimensional (1D) interfaces. These models generate

self-affine interfaces with scaling exponents given by the TKPZ equation, namely $\beta = 1/3$, $\chi = 1/2$, and $z = \chi/\beta = 3/2$ [13].

It has been shown that at approaching the depinning transition with $F \to F_c$, where the noise is quenched in nature, different models may fall into distinct universality classes [14,15,11,6]. In one of the cases, known as the directed percolation depinning (DPD) universality class, an anisotropy is generated in the system and the coefficient of the nonlinear term $\lambda \to \infty$ as the depinning transition is approached [14]. In the limit of a nonmoving interface, the scaling exponents can be related to those of directed percolation, with $\beta = \chi = 0.633$. However, the moving interface is not self-affine and the exponents are different [16]. In the isotropic universality class, the system remains isotropic at the depinning transition and consequently $\lambda \to 0$ in Eq. (1) corresponding to the quenched Edwards-Wilkinson (QEW) equation. The scaling exponents in this case are $\beta \approx 0.88$ and $\chi \approx 1.25$ [16]. It has been recently shown [6] that for isotropic systems there exists a special case where the propagation of the front couples to an underlying *isotropic* percolation (IP) transition. In this *isotropic percolation depinning* (IPD) case, the anisotropy also vanishes but now the scaling exponents are given by those of the IP transition, with $\beta = 1/d_{min} \approx 0.88$ and $\chi = 1$, where d_{min} is the minimum path exponent. In this case the description of Eq. (1) actually breaks down and the interface at the threshold is no longer self-affine.

When modeling kinetic roughening phenomena in disordered systems, the underlying medium (noise) is usually assumed to be uniformly random without long-range correlations. However, it is often the case that there exist nontrivial correlations up to some cutoff length scale ξ_p. In particular, it has been shown that in some cases spatial correlations in random media follow to a good degree of approximation a power-law type of behavior [17–20], and ξ_p may be consid-

―――――
*Author to whom correspondence should be addressed. Electronic address: Tapio.Ala-Nissila@hut.fi

erably larger than the microscopic lengths in the system. The way to at least approximately include the effect of spatial or temporal correlations of the background in the dynamics is to assume correlated noise in the corresponding continuum equation. The case of the KPZ equation with this kind of situation has been studied in many works [21–30]. Results from these studies have been somewhat inconclusive, in particular when the spatial and temporal parts of the noise autocorrelations follow power-law behavior.

In this paper we report the results of extensive numerical simulations of 2D cellular automata models where an interface propagates in an algebraically correlated random medium. The two different cases studied here include the nearest-neighbor (NN) ''forest fire'' model [3,6] where the IPD scenario can be realized at the percolation threshold c^*, and the simple DPD lattice model [31] with un underlying DP transition. We consider interface dynamics in these models both well above and close to their corresponding thresholds. Our results show that well away from the transition, the asymptotic dynamics of an interface in both models is found to be consistent with the spatially power-law correlated Kardar-Parisi-Zhang (PKPZ) universality class. In particular, we find behavior in agreement with the predictions of Medina et al. [21] for the growth exponent β [30]. At the transition, the models display different behavior. In both cases, the scaling exponents can be related to those of the underlying percolation transition similar to the case of a uniformly random background. For the IPD model, the anisotropy vanishes at the percolation threshold and the model belongs to the algebraically correlated IP universality class, whereas in the DP case the nonlinearity diverges at the percolation threshold and the exponents for the nonmoving interface are given by those of the algebraically correlated DP universality class. This implies that for both models, the exponent β increases with the strength of correlations. For the IPD model the roughness exponent $\chi = 1$ while for the DPD case $z = 1$ independent of the background correlations. Our numerical results are in good agreement with theory.

The rest of this paper is organized as follows. In Sec. II the models are introduced and the known results for uniformly random media are summarized briefly. In Secs. III and IV the results of Monte Carlo simulations for the two models in algebraically correlated random media are presented. Finally, in Sec. V we conclude and discuss our results.

II. MODELS

In this section we review the models and the results from earlier studies. We consider 2D ''forest fire'' (or ''imbibition'') types of models defined on a square lattice of size $L \times L$ with periodic boundary conditions in the x direction and free boundary conditions in the y direction. The status of a given lattice site in the models can be one of the following: (i) an empty site, (ii) a site occupied by an unburned tree, (iii) a site occupied by a burning tree, or (iv) a site occupied by a burned tree. Initially, a fraction $c \equiv \langle c(x,y) \rangle$ of the sites is occupied by trees. In the present case, the positions of the trees are determined using an algorithm that generates spatial long-range power-law-type correlations [32], so that $\langle [c(r) - c][c(0) - c] \rangle \propto r^{-\rho/2}$ up to $\xi_p \approx L$. The parameters that

characterize the models are thus the size of the system L, the average concentration c, and the exponent of the density-density autocorrelation function ρ.

The dynamics of the isotropic percolation depinning model is defined by the following set of rules: During one time step, each burning tree ignites all the nearest-neighbor unburned trees and then becomes a burned tree. Once a tree has become a burnt one, it does not change its status anymore. Also, an initially empty lattice site remains empty throughout the simulation. The spreading rules themselves are isotropic without any specific growth direction. We define the position of the interface $h(x,t)$ at column x to be given as the location of the highest burning or burnt tree in each column. We note that this definition is sufficient to make the interface single-valued. Initially, the lattice sites in the bottom row ($y = 0$) are given the status of a burning tree and the system evolves according to the aforementioned rules.

The spreading rules for the directed percolation depinning (DPD) model are almost the same as the rules for the IPD model. The important difference is that at each time step each burning tree ignites all its nearest-neighbor unburned trees and all unburned trees and empty sites in the same column below the new burning trees. This so-called erosion of the overhangs process makes the model anisotropic and changes the universality class at the depinning transition. It also makes the interface single-valued. The position of the interface is defined in the same way as in the IPD model [31].

III. CHARACTERIZATION OF INTERFACE ROUGHENING

In order to quantitatively characterize the kinetic roughening of the interface, we have considered the following quantities [12,33]. First, the global width $w(c,t,L)$ of the interface is defined by

$$w^2(c,t,L) \equiv \overline{\langle [h(\vec{r},t) - \overline{h(\vec{r},t)}]^2 \rangle}, \qquad (2)$$

where the overbar denotes a spatial average over the system of size L, and brackets denote configuration averaging. Correspondingly, the local width of the interface $w_\ell(c,t)$ can be defined as

$$w_\ell^2(c,t) \equiv \langle \langle [h(\vec{r},t) - \langle h(\vec{r},t) \rangle_\ell]^2 \rangle_\ell \rangle, \qquad (3)$$

where the notation $\langle \rangle_\ell$ now denotes spatial averaging over all subsystems of size ℓ of a system of total size L [34]. For growing self-affine interfaces, both the global and local widths satisfy the Family-Viscek scaling relation [35] and have asymptotic behavior given by

$$w(t,L) \sim \begin{cases} t^\beta & \text{for } t \ll L^z, \\ L^\chi & \text{for } t \gg L^z, \end{cases} \qquad (4)$$

and correspondingly for $w_\ell(t)$. The quantities β and χ define the growth and roughness exponents, respectively, and $\chi = \beta z$. We note that in addition to using the width, scaling exponents can be obtained by using the height-height difference correlation function,

$$C(r,t) = \overline{\langle [\delta h(\vec{r}_0, t_0) - \delta h(\vec{r}_0 + \vec{r}, t_0 + t)]^2 \rangle}, \qquad (5)$$

with $\delta h \equiv h - \bar{h}$, in the appropriate regimes [12].

In some cases the interfaces may exhibit multiscaling [36,37]. This is revealed by the existence of different scaling exponents for different moments of the height distribution. Furthermore, some models may show anomalous scaling in the local width [33,37]. In this case, the scaling of $w_{\swarrow}(t)$ is given by

$$w_{\swarrow}(t) \sim \begin{cases} t^{\beta} & \text{for } t \ll \ell^z, \\ \ell^{\chi_{loc}} t^{\beta_*} & \text{for } \ell^z \ll t \ll L^z, \\ \ell^{\chi_{loc}} L^{z\beta_*} & \text{for } \ell^z \ll L^z \ll t, \end{cases} \qquad (6)$$

where χ_{loc} is the local roughness exponent and $\beta_* = (\chi - \chi_{loc})/z$. Corresponding scaling forms can also be written for the correlation functions [33]. This behavior was demonstrated recently for the QEW equation, with $\chi_{loc} = 1$ and $\beta_* \approx 0.21$ [38].

IV. ISOTROPIC AND DIRECTED PERCOLATION DEPINNING MODELS IN A UNIFORMLY RANDOM BACKGROUND

It is well established that for $F \gg F_c$ the quenched noise in Eq. (1) can be replaced by thermal noise for scales much larger than the size of the pinned regions [39]. The corresponding TKPZ exponents have been numerically verified for both the DPD [12] and the IPD [6] models. However, in the vicinity of the percolation transition the corresponding correlation length diverges leading to different behavior. In the IP case, we have previously shown [6] that the continuum description of Eq. (1) breaks down close to $c = c^* \approx 0.59275$ and the scaling exponents for $\xi_{\|}(t) \ll \xi(c)$ are directly related to those of the IP transition [$\xi_{\|}(t) \sim t^{1/z}$ and $\xi(c)$ denote the lateral correlation length of the moving interface and the percolation correlation length, respectively]. The global scaling exponents are given by $\chi = 1$ and $z = d_{min} \approx 1.13$, where d_{min} is the minimum path exponent. In addition, the IPD model shows multiscaling and anomalous scaling of the local width. Close to the transition, the average velocity of the interface vanishes as

$$v(c) \sim (c - c^*)^{\theta}, \qquad (7)$$

and we estimated the velocity exponent to have the value $\theta \approx 0.17$. This is in agreement with the scaling relation $\theta = (z - \chi)\nu$, where $\nu = 4/3$ is the IP correlation length exponent.

For the DPD model the situation is somewhat different. At the DP transition $c^* \approx 0.53$ [10] there are two correlation lengths that behave as

$$\xi_{\|}^D \sim |c - c^*|^{-\nu_{\|}} \quad \text{and} \quad \xi_{\perp}^D \sim |c - c^*|^{-\nu_{\perp}}, \qquad (8)$$

and the scaling exponents of the *nonmoving interface* are given by the ratio of the perpendicular exponent to the parallel one, i.e., $\chi = \beta = \nu_{\perp}/\nu_{\|} \approx 0.633(1)$ and $z = 1$ [10,11,40]. The corresponding velocity exponent is $\theta \approx 0.64$ [16,41]. The moving interface close to c^*, however, is not self-affine and the exponents change. The exponents β and χ as determined from $w(t,L)$ are about $0.7 - 0.8$ [7,11,16,42].
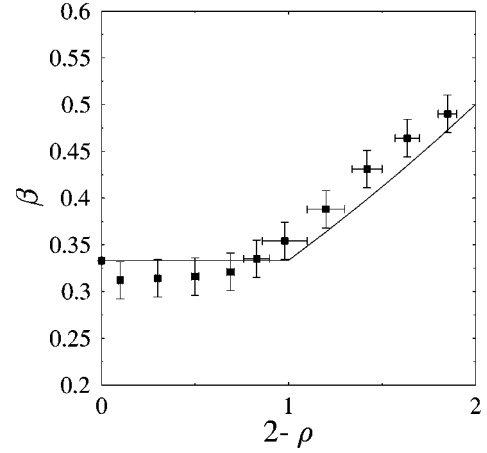

FIG. 1. The growth exponent $\beta(\rho)$ vs ρ for the IPD case well above c^* at $c = 0.99$ [49]. The data are for 2048×2048 systems averaged over 500 configurations. The straight line shows the prediction of Medina *et al.* [21] for the PKPZ equation.

V. THE CASE OF ALGEBRAICALLY CORRELATED BACKGROUND

In this section we present the results of extensive numerical work on the IPD and DPD lattice models with an algebraically correlated spatial distribution of reactants. In this case the two-point correlation function asymptotically satisfies

$$\langle c(\vec{r}) c(\vec{r}') \rangle \sim (\Delta r)^{-\rho/2} \equiv [(\Delta x)^2 + (\Delta y)^2]^{-\rho/u}, \qquad (9)$$

where $\Delta x = |x - x'|$, $\Delta y = |y - y'|$, and $\Delta r = |\vec{r} - \vec{r}'|$, and $c(x,y)$ denotes the local density of reactants.

A convenient method for generating an algebraically correlated distribution on a lattice is the so-called modified Fourier filtering method [32], which allows one to vary the exponent ρ for a given c. This method allows one to generate power-law-type correlations extending through most of the system, i.e., the cutoff $\xi_p \sim L$. A drawback of this method is that the correlations have to be generated for the whole lattice initially. This severely limits the maximum system sizes available for simulations. In our case, the systems were typically of size 2048×2048.

A. IPD and DPD models away from the percolation threshold

Let us first discuss our results in the limit of relatively high concentrations, i.e., when $\xi(c) \ll \xi_{\|}(t) \ll \xi_p$. In the case of the IPD model we find that as ρ is varied between 2 and 0, the growth exponent β depends on ρ. In Fig. 1 we show the numerical results for β vs $2 - \rho$. First, we note that β is approximately a constant ($\approx 1/3$) for $2.0 \gtrsim \rho \gtrsim 1.3$, and that it increases for decreasing ρ for $0 < \rho \lesssim 1.0$.

Our numerical estimates for $\beta(\rho)$ agree well with the renormalization-group calculations of the KPZ equation with algebraically correlated noise by Medina *et al.* [21,30]. This may not be surprising, since it is conceivable that an algebraically correlated reactant field should translate into an algebraically correlated effective noise at the interface. Using a simple geometric construction, it can be argued that the effective noise $\eta(\vec{r})$ at the interface satisfies
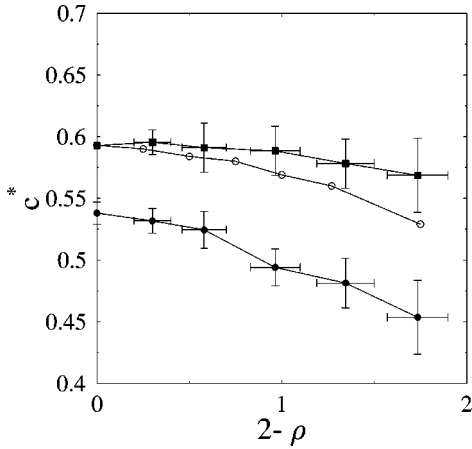
FIG. 2. Percolation thresholds c_{IP}^* (filled squares) and c_{DP}^* (filled circles) vs ρ for the IPD and DPD models as obtained from the scaling of the interface velocity $v(c)$. The results from Ref. [43] are shown with open circles. See text for details.

$$\langle \eta(\vec{r}) \eta(\vec{r}') \rangle \sim (\Delta r)^{-\rho/2} \sim [(\Delta x)^2 + (\Delta h)^2]^{-\rho/u}. \quad (10)$$

Let us next make the assumption that the slopes are small, i.e., that $\Delta h \ll \Delta x$. Then it is plausible that the Δh term can be neglected. These assumptions lead to an effective noise at the interface given by

$$\langle \eta(\vec{r}) \eta(\vec{r}') \rangle \sim |x - x'|^{-\rho/2}. \quad (11)$$

Furthermore, it is plausible that derivative terms in the effective equation of motion for the interface arise from the microscopic ignition rules, and therefore they should be the same as in the uncorrelated nearest-neighbor case. Therefore, the leading effect of the spatially correlated reactant distribution is to cause the noise correlations at the interface to be algebraically correlated in space. Hence, we expect the interface to display dynamics in accordance with the algebraically correlated KPZ universality class, in agreement with numerical results.

We note that similar arguments apply for the DPD case, too. To this end we have simulated the DPD model for $\rho = 0.98(12)$ and find $\beta = 0.37(3)$, and $\rho = 0.15(5)$ and find $\beta = 0.48(2)$, in agreement with the behavior of the IPD model.

B. IPD and DPD models at the percolation threshold

Let us next discuss the behavior of the interface in the vicinity of $c = c^*(\rho)$. In Fig. 2 we show c_{IP}^* and c_{DP}^* versus ρ as obtained by calculating the velocity $v(c)$ of the interface and finding $c^*(\rho)$ and $\theta(\rho)$ such that v displays the best scaling according to $v \sim [c - c^*(\rho)]^{\theta(\rho)}$. We note that the value of the percolation threshold depends on the strength of the correlations $c^* = c^*(\rho)$ and it is not known very accurately [43]. The values of the velocity exponent obtained by using this method are shown in Fig. 3. The value of θ seems to decrease slowly as ρ goes from $2 \to 0$. Our results are in good agreement with Prakash et al. [43] for the IPD case but the system sizes here are much larger than in their study. For
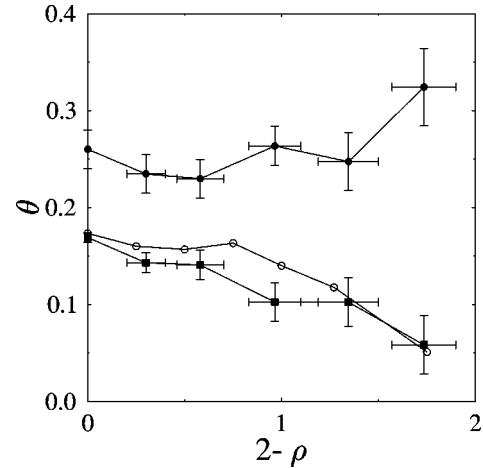

FIG. 3. Velocity exponents $\theta(\rho)$ vs ρ for the IPD (filled squares) and DPD (filled circles) models. Our results are close to the data of Prakash et al. [43] shown with open circles.

the DPD case, however, we find a value of θ at $\rho = 2$ (uncorrelated background) much lower than those reported in the literature [11,16,41].

Again, we can relate the exponents characterizing the interface to the ones characterizing the critical percolation cluster. Let us first discuss the isotropic case. First, due to the isotropy of the system, for $\xi_\parallel(t) \ll \xi(c)$ we expect $\chi(\rho) = \chi = 1$ for the global roughness exponent. Second, a previous study of the correlated site isotropic percolation problem [43] revealed that although the fractal dimension of the percolation cluster does not depend on ρ, the minimum path exponent is a function of ρ. In other words, using the argument that $z = z(\rho) = d_{min}(\rho)$ and $\chi = 1$, it follows that $\beta = \beta(\rho)$. In particular, β should increase as $\rho = 2.0 \to 0.0$ [43].

Since $c^*(\rho)$ is not known very accurately, we estimated the growth exponent $\beta(\rho)$ with several different concentrations near the approximate percolation threshold. Then we chose our best estimate to be the value of β that displayed the longest scaling region and had the largest value [44]. In Fig. 4 we show the results of our simulations for 8192×512 lattices averaged over 200 configurations. The value of the growth exponent depends clearly on the strength of the correlations and increases as $\rho = 2.0 \to 0.0$, as expected. The first point, $\rho = 2$, has been taken from our previous study [6] and has smaller errors than the other points in Fig. 4. In Fig. 4 we also show results obtained using the values of d_{min} obtained from Ref. [43], where $L = 104$.

The separate point in Fig. 4 marked with a diamond is the result from a simulation where we used the 2D square lattice Ising model to produce the long range correlations in the lattice. Namely, exactly at the critical temperature T_c the Ising model generates long range correlations in the lattice with an exponent $\rho = 0.25$ [45]. This method is applicable for the IPD model at c^* because the critical point is also the percolation point of the lattice [46]. The value of $\beta(L = 1000)$ obtained by this method is consistent with our other estimates.

For the DPD model, we applied the same method of Ref. [32] for generating the power-law correlations in the lattice. The same methods as in the IPD case were used to find the
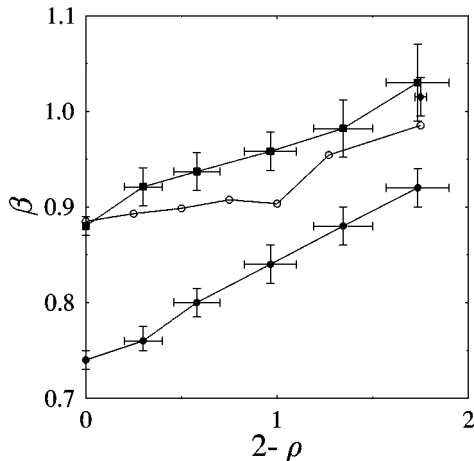
FIG. 4. Growth exponents $\beta^*(\rho)$ vs ρ at the depinning transition for the IPD (filled squares) and DPD (filled circles) models. The value obtained using the 2D Ising configuration at T_c corresponding to $\rho = 0.25$ is shown by a diamond. Values of $\beta(\rho)$ obtained from the data of Ref. [43] are shown by open circles.

growth exponents $\beta(\rho)$ and the velocity exponents $\theta(\rho)$ for the moving interface. The results from our simulation are shown in Figs. 3 and 4. The value of the growth exponent $\beta(\rho)$ increases almost linearly as ρ goes from $2 \rightarrow 0$, and $\beta(\rho = 2)$ (uncorrelated case) is in good agreement with the estimates reported in the literature [7,12,11,16]. The data for the velocity exponent $\theta(\rho)$ are not sufficiently good to determine the behavior of $\theta(\rho)$ very accurately. The high value of the last point may indicate that the velocity exponent $\theta(\rho)$ increases as ρ goes from $2 \rightarrow 0$.

We assume that the arguments leading to the relation $z = d_{\min}$ [11,47] are valid also in the correlated case. In one dimension the minimum path exponent for the DPD model equals unity independent of spatial correlations. Thus, in the correlated case $z = 1$ also, and $\chi(\rho) = \beta(\rho)$. Assuming that the scaling relation $\theta(\rho) = \nu_\parallel(\rho)[z - \chi(\rho)]$ holds, our data can be used to estimate the parallel correlation length exponent $\nu_\parallel$ as a function of ρ.

VI. SUMMARY AND CONCLUSIONS

In this work we have studied the dynamics of interfaces in algebraically correlated random media through Monte Carlo simulations of two different lattice cellular automata model. These include the two cases where the pinning of the emerging interface is due to either an isotropic or a directed percolation transition of the underlying lattice. We find that well above the percolation transition, the dynamics is consistent with the spatially power-law correlated Kardar-Parisi-Zhang (PKPZ) [21,30] universality class for both models. At the percolation transition, however, we find different behavior for the two cases. The exponents that characterize the interfaces at the transition are related to the exponents of the percolation cluster in analogy to the uniformly random case [6,11]. In the present case this means that for the IPD case, $\chi = 1$ but z and thus also β depend on ρ. Our numerical estimates of these exponents are in reasonably good agreement with other works [43]. For the DPD model, $z = 1$ independent of the correlations while β and χ depend on ρ and should be equal to each other.

An interesting physical example where the scenario of a power-law correlated background could be realized is that of dynamics of slow combustion in paper [2,7]. Through measuring the actual density distributions of laboratory manufactured paper sheets, it has been demonstrated that very low density paper may exhibit a power-law type of density correlations that exist up to about 15 times the fiber length, i.e., the scale of a few cm [20]. In the early experiments of Zhang *et al.*, where low-density paper was burned [2], a value of the roughness exponent was reported in agreement with the DPD case. The new experimental results of Maunuksela *et al.* [7] on higher density paper show that asymptotically the exponents are consistent with the TKPZ case. It would be of great interest to study slow combustion using paper with well-characterized intermediate power-law correlations in density, since the PKPZ scenario could perhaps be realized there.

[1] G. Blatter, M. V. Feigelman, V. B. Geshkenbeim, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1995).

[2] J. Zhang, Y.-C. Zhang, P. Alstrøm, and M. T. Levinsen, Physica A **189**, 383 (1992).

[3] G. Albinet, G. Searby, and D. Stauffer, J. Phys. (Paris) **47**, 1 (1986).

[4] N. Provatas, T. Ala-Nissila, M. Grant, K. R. Elder, and L. Piché, Phys. Rev. E **51**, 4232 (1995).

[5] N. Provatas, T. Ala-Nissila, M. Grant, K. R. Elder, and L. Piché, J. Stat. Phys. **81**, 737 (1995).

[6] M.-P. Kuittu, M. Haataja, N. Provatas, and T. Ala-Nissila, Phys. Rev. E **58**, 1514 (1998); *ibid.* (to be published).

[7] J. Maunuksela, M. Myllys, O.-P. Kähkönen, J. Timonen, N. Provatas, M. J. Alava, and T. Ala-Nissila, Phys. Rev. Lett. **79**, 1515 (1997); L. A. N. Amaral and H. A. Makse, *ibid.* **80**, 5706

(1998); J. Maunuksela, J. Myllys, J. Timonen, T. Ala-Nissila, M. Kuittu, M. J. Alava, and N. Provatas, *ibid.* **80**, 5707 (1998).

[8] V. K. Horváth, F. Family, and T. Vicsek, J. Phys. A **24**, L25 (1990).

[9] S. He, G. L. M. K. S. Kahanda, and P.-z. Wong, Phys. Rev. Lett. **69**, 3731 (1992).

[10] S. V. Buldyrev, A.-L. Barabási, F. Caserta, S. Havlin, H. E. Stanley, and T. Vicsek, Phys. Rev. A **45**, R8313 (1992).

[11] L. A. N. Amaral, A.-L. Barabási, S. V. Buldyrev, S. T. Harrington, S. Havlin, R. Sadr-Lahijany, and H. E. Stanley, Phys. Rev. E **51**, R4655 (1995); L. A. N. Amaral, A.-L. Barabási, S. V. Buldyrev, S. T. Harrington, S. Havlin, R. Sadr-Lahijany, and H. E. Stanley, *ibid.* **51**, 4655 (1995); L. A. N. Amaral, A.-L. Barabási, H. A. Makse, and H. E. Stanley, *ibid.* **52**, 4087 (1995).

[12] A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, England, 1995).

[13] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).

[14] L. A. N. Amaral, A.-L. Barabási, and H. E. Stanley, Phys. Rev. Lett. **73**, 62 (1994).

[15] L.-H. Tang, M. Kardar, and D. Dhar, Phys. Rev. Lett. **74**, 920 (1995).

[16] H. Leschhorn, Phys. Rev. E **54**, 1313 (1996); H. Leschhorn, T. Nattermann, S. Stepanow, and L.-H. Tang, Ann. Phys. (Leipzig) **6**, 1 (1997).

[17] M. Sahimi, Rev. Mod. Phys. **65**, 1393 (1993).

[18] A. H. Thompson, A. J. Katz, and C. E. Krohn, Adv. Phys. **36**, 652 (1987).

[19] N. Provatas, T. Ala-Nissila, and M. J. Alava, Phys. Rev. Lett. **75**, 3556 (1995).

[20] N. Provatas, M. J. Alava, and T. Ala-Nissila, Phys. Rev. E **54**, R36 (1996).

[21] E. Medina, T. Hwa, M. Kardar, and Y.-C. Zhang, Phys. Rev. A **39**, 3053 (1989).

[22] P. Meakin and R. Jullien, Europhys. Lett. **9**, 71 (1989).

[23] P. Meakin and R. Jullien, Phys. Rev. A **41**, 983 (1990).

[24] Y.-C. Zhang, P. Meakin, and R. Jullien, Phys. Rev. B **42**, 4897 (1990).

[25] H. G. E. Hentchel and F. Family, Phys. Rev. Lett. **66**, 1982 (1991).

[26] H. Ma and B. Ma, Phys. Rev. E **47**, 3738 (1993).

[27] C.-K. Peng, S. Havlin, M. Schwartz, and H. E. Stanley, Phys. Rev. A **44**, R2239 (1991).

[28] J. G. Amar, P.-M. Lam, and F. Family, Phys. Rev. A **43**, 4548 (1991).

[29] P.-M. Lam, L. M. Sander, and D. E. Wolf, Phys. Rev. A **46**, R6128 (1992).

[30] H. K. Janssen, U. C. Täuber, and E. Frey, cond-mat@xxx.lanl.gov preprint 980825. In this recent work, exact exponents are obtained for the PKPZ case as in Medina *et al.* [21].

[31] Our model is a simple variant of imbibition model of [10] or the same as in [48] without a gradient in the strength of the disorder in the underlying lattice.

[32] H. A. Makse, S. Havlin, M. Schwartz, and H. E. Stanley, Phys. Rev. E **53**, 5445 (1996).

[33] J. M. López and M. A. Rodríguez, Phys. Rev. E **54**, R2189 (1996); J. M. López, M. A. Rodríguez, and R. Cuerno, *ibid.* **56**, 3993 (1997).

[34] Alternatively, one can use the qth moments of the average nearest-neighbor height differences $\sigma_q(t) = \langle |h(x+1,t) - h(x,t)|^q \rangle^{1/q}$ [37].

[35] F. Family and T. Vicsek, J. Phys. A **18**, L57 (1985).

[36] A.-L. Barabási, R. Bourbonnais, M. Jensen, J. Kertész, T. Vicsek, and Y.-C. Zhang, Phys. Rev. A **45**, R6951 (1992).

[37] J. Krug, Phys. Rev. Lett. **72**, 2907 (1994).

[38] J. M. López and M. A. Rodrígues, J. Phys. I **7**, 1191 (1997).

[39] T. Nattermann, S. Stepanov, L.-H. Tang, and H. Leschhorn, J. Phys. II **2**, 1483 (1992).

[40] L.-H. Tang and H. Leschhorn, Phys. Rev. A **45**, R8309 (1992).

[41] We did independent simulations of the DPD model studied here in the uniformly random case, but were unable to reproduce this value of θ. Using a direct least-squares fitting to $v(c)$ close to c^* gave $\theta \approx 0.26$. Using the same method as Amaral *et al.* in Refs. [11] produced $\theta \approx 0.424$.

[42] We calculated the first four moments of $\sigma_q(t)$ [34] for the DPD model at c^* (moving interface). We saw no evidence of multiscaling behavior and all moments seem to saturate very fast without any clear power-law scaling region.

[43] S. Prakash, S. Havlin, M. Schwartz, and H. E. Stanley, Phys. Rev. A **46**, R1724 (1992).

[44] It is reasonable to assume that the roughening of the interface is fastest exactly at the percolation threshold, and that the scaling regime for power-law behavior is longest there.

[45] R. J. Baxter, *Exactly Solved Models in Statistical Mechanics* (Academic Press, London, England, 1982).

[46] A. Coniglio, C. R. Nappi, F. Peruggi, and L. Russo, J. Phys. A **10**, 205 (1977).

[47] S. Havlin, L. A. N. Amaral, S. V. Buldyrev, S. T. Harrington, and H. E. Stanley, Phys. Rev. Lett. **74**, 4205 (1995).

[48] L. A. N. Amaral, A.-L. Barbási, S. V. Buldyrev, S. Havlin, and H. E. Stanley, Phys. Rev. Lett. **72**, 641 (1994).

[49] We chose a very high reactant concentration in order to minimize the crossover effects caused by the percolation transition.